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ON THE "RESOLVING CAPACITY" OF SYSTEMS MEASURING THE HORIZONTAL DIMENSIONS OF IONOSPHERIC HETEROGENEITIES

[Following is a translation of an article written by V. D. Gusev, S. F. Mirkotan, Yu. V. Berezin and M P. Kiyanovskii in Vestnik **Moskovskogo Universiteta, No. 4, 1959, pages 105-115.**]

Introduction

In investigation of heterogeneities in the structure of the ionosphere and the broad distribution of motion in it we came upon the so-called spatially dispersed method [1] . Observations at three points of any parameter of a radio signal reflected from the ionosphere (amplitude, angle of entry, group or phase path) enable obtaining, in particular, knowledge of the spectrum of horizontal dimensions of heterogeneous formations and of the rates of their movements in the ionosphere [1, 2]. Three points located on the surface of the earth form the measuring triangle. This triangle is a constituent part of a measuring system which possesses limited possibilities as regards the determination of the various dimenstions of the heterogeneities, that is, the spectrum of the dimensions measurable by means of a given system is limited. It should be taken into account here that in the concept of a measuring system there is included not only the relative position of the observation points but the method of processing experimental material, since peculiarities of the method can exert a decisive influence in determining the possibilities of a measuring system.

The following questions arise in the allocation of observations points: what should the distance (base) be between observation points; how is the "measuring triangle", consisting of three observation points, to be orientated with regard to the countries of the world; which dimensions of the heterogeneities can be measured on the bases selected; what will be the accuracy of the determination of various dimensions of the heterogeneities at the base selected. These questions have not received due explanation in the literature up to now, although they play a very serious role in preparing the experiment and in evaluating the reliability of the results of research on the heterogeneous structure of the ionosphere. In the present work we will attempt to give answers to the above questions.

The method of investigation has a general character, that is, is comparable for both large and small scale research on heterogeneities of the ionosphere. Concrete evaluations have been made in the instance of investigation of large-scale heterogeneities of the F_2 layer of the ionosphere.

APPARATUS AND METHOD OF PROCESSING

THE EXPERIMENTAL MATERIALS

The phase method of registering large heterogeneities [4] was used in making the experimental investigations.

Variations in the phase path φ (t) of the signal from the F_2 layer were measured. The indicated variations in the phase path of the reflected signal are caused by heterogeneous formations in the ionosphere. The principle of the change in variation of the phase path of the signal can be explained by the following block-system. (Fig. 1). The probing of the atmosphere is conducted with impulses of t_{rep} = 50 hertz and duration t = 150-200 maxwells. A wide-band amplifier assures a pulse power of 2 kilovolts. The high-frequency voltage of the master oscillator acts simultaneously on the modulable power amplifier and the system of phase comparison. The reflected high-frequency signal, amplified by the receiver and converted to an intermediate frequency is given to the phase comparison system

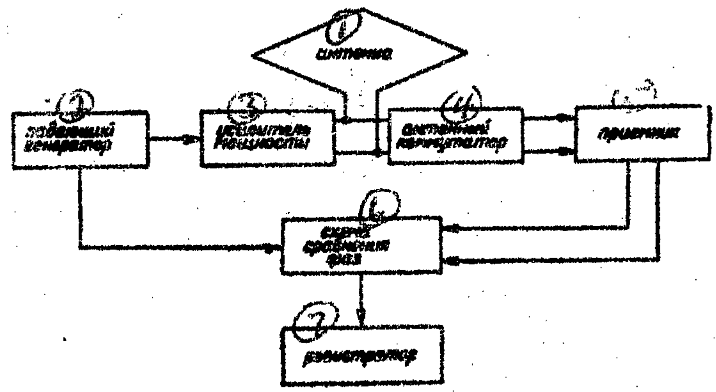

Fig. 1. Block-system of arrangement for recording variations in the phase path of a reflected signal. 1) antenna; 2) master oscillator; 3) power amplifier; 4) antenna switch; 5) receiver; 6) phase comparison system; 7) recorder.

(phase meter). In addition, the voltage of a local heterodyne is contributed from the receiver on the phase meter, and by means of it the high-frequency voltage of the master oscillator is converted into reverse voltage of intermediate frequency. The phase of the reflected signal is compared in the phase meter with that reverse voltage. With change of the phase path of the reflected signal its phase is changed relative to the phase of the reverse voltage and these changes are registered by the recording device on film[5].

The measurements were made in the region of Moscow. Three observation points, equipped with apparatus similar to that depicted in Figure 1 were placed so that they formed a triangle with sides of 28.43 and 62 km (Fig. 2). The observation interconnected by a system of two-channel decimetric radio circuits. The work of the three observation points is synchronized in time with minute and hour signals given on one of the two channels of the radio circuit. The second channel is used for official conversations.

Regular measurements of the parameters of the large scale heterogeneities and their movements were begun in May, 1956, and are being conducted at the present time according to the program of the International Geophysical Year. Observation is made at all three points on one frequency (f_v = 2 - 7 megahertz), selected as the central point in conformity with ionospheric conditions.

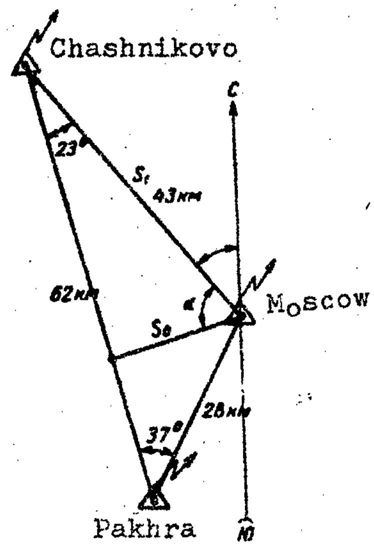

Fig. 2. Disposition of the observation points, the "measuring triangle".

The results of the experiment are registered by the recorder on motion picture film. A sample of the record is given in Figure 3. Each "tooth" represents a change of 2π in the phase path of the reflected signal. Points A are the points of the relative extremes of the phase path.

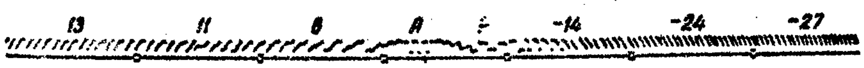

Fig. 3. Specimen of recording of variations in phase path.

The records of the variation in φ (t) at all three observation points are more or less the same, but one is displaced relative to the other and this demonstrates the motion of the heterogeneities.

The results of the measurements are subjected to statistical processing: the duration of the continuous record of variations in the phase path should assure that it is acceptable from the statistical point of view . Therefore observations with a duration of not less than four hours are subjected to processing. The operating frequency of the transmitters at the three observation points does not change during this time interval.

The autocorrelation functions $\rho_u(\tau)$ for each observation point and the mutual correlation functions $\rho_{ij}(\tau)$ for each pair of observation points are calcu-

lated from the records of variations in phase path of reflected signals.

The following values in particular are determined from the graphs of these functions: the temporal characteristic of the observed random process $\tau_{0.5}$ in which $\rho_{ij}(\tau_{0.5}) = 0.5$; the "characteristic" time in which $\rho_{ij}(0) = \rho_{ii}(\tau_S)$; the value of $\rho_{ii}(\tau_S)$, which can be further designated as P_S.

Using the values mentioned and the known distances of S between the observation points (base lines), we can determine in particular the horizontal dimensions of the heterogeneities:

$$\Delta = 4\tau_{0,5} \cdot V'_c = 4S \cdot \frac{\tau_{0,5}}{\tau_S}, \tag{1}$$

where V'_c is the "characteristic velocity": $V'_c = S/\tau_S$. A complete explanation of the method of processing the experimental results is given in the literature [6].

"Resolving Capacity" of the Measuring System and Accuracy of Measurement of the Horizontal Dimensions of the Heterogeneities

The maximum and minimum dimensions which can be measured by a system with a given accuracy and probability will in the future be called its "resolving capacity". Under real conditions the observation interval T of a random process $\varphi(t)$ always is limited. This fact also imposes limitations on the possibilities of the measuring system we are discussing.

Thus, due to the limitedness of the interval T, the experimental correlative functions of the process $\rho(\tau)$ will differ from the theoretical (at an infinite interval) that is, will contain a statistical error:

$$\rho_{\text{теор}}(\tau) = \rho_{\text{эксп}}(\tau) \pm \delta\rho, \tag{2}$$

$$\delta\rho = \alpha\sigma_\rho. \tag{3}$$

If $\varphi(t)$ obeys the normal law of distribution, then

$$\sigma_\rho \simeq \frac{1-\rho^2}{\sqrt{N-1}}, \tag{4}$$

where $N = T/2\tau_{0.5}$ is a number determining the extent of choice. Expression (4) is strictly comparable in the case of a normal distribution of values of $\rho(\tau)$, which occurs at N 50 and values of $\rho(\tau)$ not very close to unity. For $N < 50$ or $\rho(\tau)$ close to unity the Fisher conversion allows finding the value of σ_ρ.

It has been established [7] that expression (4) for σ_ρ can be used as a first approximation for an arbitrary distribution of $\varphi(t)$ 8 -- at any rate, for distributions with which one must deal in investigations of heterogeneities of the ionosphere. In investigations of ionospheric heterogeneities the correlation function $\rho(\tau)$ usually has the form of an evenly diminishing function (Fig. 4, a and b). For these investigations it is sufficient to know the behavior of the function $\rho(\tau)$ in the region of the values $(-\tau_1; +\tau_1)$ near the maximum $\rho(\tau)$, where it diminishes from unity to a small enough value. This is the region of the main values of function $\rho(\tau)$. In the region of the main values $\rho(\tau)$ can be approximated with a sufficient degree of accuracy by the polynom:

$$\rho(\tau) = 1 + a\tau + b\tau^2 + \ldots$$

Since $\rho(\tau)$ is not determined accurately, all the parameters of the heterogeneities determined by means of the correlative functions will also contain a statistical error.

In the present work the error is considered only in determining the horizontal dimensions of heterogeneities. The "resolving capacity" of the measuring system depends on this error, and likewise the accuracy of determination of the dimensions of the heterogeneities.

As has already been indicated [1], a horizontal dimension of a heterogeneity can be expressed by temporal characteristics of the observed random process and the distance S between the observation points:

$$\Delta = 4S\frac{\tau_{0.5}}{\tau_s}.$$

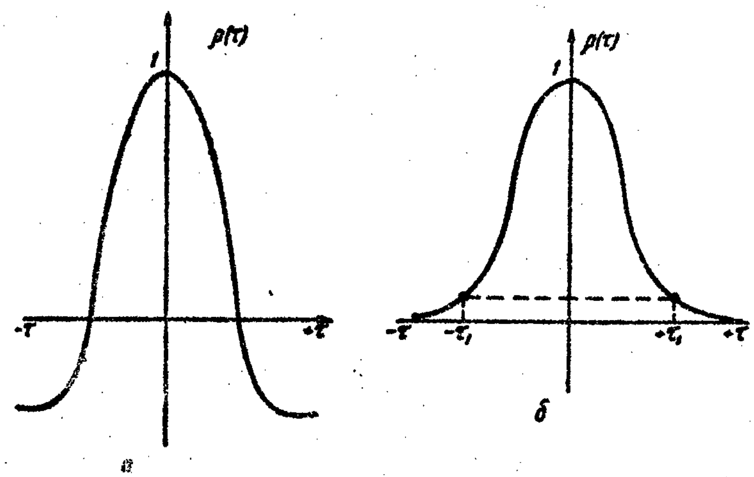

Fig. 4. Possible forms of the correlation functions. a - the function attains its own zero value; b - the function approaches zero asymptotically.

To determine the relative error $\delta\Delta/\Delta$ we assume that the correlative function can be approximated by a parabola: $\rho(\tau) = 1 - a\tau^2$. Then the values $\tau_{0.5}, \tau$ and ρ_s will be linked with one another by the functional dependence

$$\rho_S = f(\tau_{0,5};\ \tau_s).$$

Since, according to (1), Δ is a function of the ratio $\tau_{0.5}/\tau_s$,

$$\Delta = \psi(\tau_{0,5}/\tau_s),$$

Δ will be a function of the value ρ_s, that is, $\Delta = \phi(\rho_s)$. Consequently, the error of $\partial\Delta/\Delta$ is

$$\frac{\delta\Delta}{\Delta} = \frac{\varphi'}{\varphi}\delta\rho_S.$$

The form of the function $\phi(\rho_s)$ will depend on the form of the approximating function.

In the case of the assumed approximation

$$\rho_s = f(\tau_{0,5};\ \tau_s) = 1 - \frac{1}{2}\left(\frac{\tau_s}{\tau_{0,5}}\right)^2$$

$$\Delta = \varphi(\rho_s) = \frac{4S}{\sqrt{2(1-\rho_s)}}. \qquad (5)$$

The relative error of $\partial\Delta/\Delta$ will be determined by the following expression

$$\frac{\delta\Delta}{\Delta} = \frac{\delta\rho_S}{2(1-\rho_s)}. \qquad (6)$$

As a result of the fact that the value of τ_s is from the equality

$$\rho_{ij}(0) = \rho_{ii}(\tau_s),$$

where each function is determined with an error on account of the finiteness of the selection, the expression (6) obtained is not accurate and should contain one more multiple $1 \leqslant c \leqslant 2$. The case $c = 2$ corresponds to complete independence of errors $\delta\rho_{ij}(\tau)$ and $\partial\rho_{ij}(\tau)$. The case $c = 1$ implies complete dependence between them. In the future $c = 1$ is assumed. Using (3) and (4), we have

$$\frac{\delta\Delta}{\Delta} \cdot \frac{\alpha(1+\rho_s)}{2\sqrt{N-1}} = \gamma. \qquad (7)$$

Relationships (5) and (7) enable answering the question posed in the introduction to this paper.

Relationship (5) indicates that the maximum dimension Δ_{max} corresponds to the maximum values of ρ_s, and the minimum dimension Δ_{min} to the minimum ρ_s.

It follows from analysis (7) that with a fixed extent of choice N and an assumed probability $\rho(\alpha)$ the values of ρ_{max} are limited from above by the given accuracy of measurement γ, but the values of ρ_{min} are not limited by the accuracy. The limitation is imposed by the region of the main values of $\rho(\tau)$, that is

$$\rho_{s\,min} = \rho(\tau_1).$$

Thus the minimum dimension of the heterogeneity which can be measured on the selected base is

$$\Delta_{min} = \frac{4S}{\sqrt{2[1-\rho(\tau_1)]}}. \qquad (8)$$

Usually $\rho(\tau_1) \leq 1$ in this case, $\Delta_{min} \cong 4S\sqrt{2} \cong 2.8S$. It should be noted that the given accuracy of measurement γ, in the determination of Δ_{min} at an assumed probability $p(\alpha)$, can be attained only in a definite extent of selection, as follows from (7).

We will proceed to the determination of Δ_{max}. The value of $\rho_{s\,max}$ at given γ and $p(\alpha)$ can be obtained by using (7):

$$\rho_{s\,max} = \frac{2\gamma\beta}{\alpha} - 1, \qquad (9)$$

where $\beta = \sqrt{N-1}$; and since $\rho_s \leq 1$, it is necessary that

$$1 \leq \frac{2\gamma\beta}{\alpha} \leq 2. \qquad (8)$$

It follows from (9) that, at given γ and $p(\alpha)$, $\rho_{s\,max}$ will correspond to β_{max}, that is, the maximum really possible extent of choice. It is clear that the extent $N = T/2\tau_{0.5}$ is determined in particular by the duration T of the observation interval.

It proved possible to limit T to four hours for phase investigations of the heterogeneous ionosphere. Further increase of this interval is inconvenient due to violations of the requirement of fixedness of the process during the course of the experiment (sharp change in the critical frequencies, etc.). Thus we will start with the fact that the maximum possible value of T is four hours.

It is likewise necessary to establish this maximum value of T under the conditions of other experimental investigations and, proceeding from it, analyze the resolving capacity of the system.

The maximum extent of selection at a given observation interval T corresponds to $\tau_{0.5\,min}$. We will recall that $\tau_{0.5}$ is the temporal characteristic of the process and is determined by it.

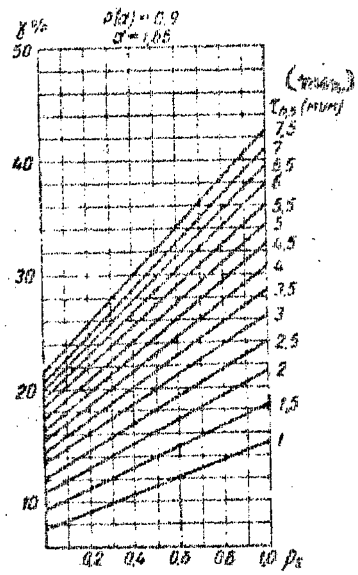

Fig. 5. Statistical error in the determination of 1 large heterogeneities as a function of the magnitude for various size volumes (in the fixed interval of observation T = 4 hours).

In the described experiment the limitation on $\tau_{0.5\,min}$ is imposed by the peculiarities of the method of processing the experimental results. The values of $\phi(t)$ are offset in a one-minute interval, and as a result values of $\tau_{0.5} < 1$ minute cannot be obtained. Consequently, the maximum extent of selection is

$$N = \frac{T}{2\tau_{0,5}} = \frac{240 \text{ мин.}}{2 \text{ мин.}} = 120$$

and

$$\beta_{max} = \sqrt{N-1} \approx 11.$$

It can be seen from expression (9) that at a given value of β_{max} the maximum values of ρ_s will be calculated with a different degree of probability and different accuracy. And since the accuracy of the results of the measurements and their probability of concept are not adequate, it does not appear possible for them to unite and form any one value which takes both factors into consideration.

Results obtained with a probability $p(a) = 0.9 (a = 1.66)$ can be considered practically certain. If we accept this probability the maximum values of ρ_s at various accuracies γ can be calculated by using (9) (see the table).

γ	8%	9%	10%	11%	12%	13%	14%	14,5%	15%
$\rho_{s\,max}$	0,06	0,19	0,32	0,45	0,59	0,72	0,85	0,92	0,98
$A_{max}S$	2,92	3,14	3,43	3,84	4,32	5,46	7,3	10,0	23,2

The accuracy $\gamma = 15\%$ can be considered acceptable for the determination of dimensions of the heterogeneities

Hence $\rho_{s\,max} = 0.98$ (see Table), and this also determines

the maximum dimension which can be measured on a given base and at a given T:

$$\Delta_{max} = 23{,}2S. \qquad (10)$$

We recall that in a normal distribution of initial values the selective law of distribution of $\rho(\tau)$ stays near normal during great changes of N ($N \geqslant 50$), even if the values of $\rho(\tau)$ are large, and it is all the closer to normal the smaller $\rho(\tau)$ is and the larger N is. In the case under discussion the law of distribution of $\varphi(\tau)$ is normal and the distribution of ρ_s can be considered normal since at $\rho_s = 0.98N$ it is large enough ($N = 120$). At $\rho_s = \rho(\tau_s) \to 0$ $N \to 43$. Consequently the estimate of the dispersion used by us, σ_ρ, is correct. Thus the estimate of the accuracy of the determination of the dimensions of ionospheric heterogeneities in each concrete experiment can be made, when the values of ρ_s and $\tau_{0,5}$ are known, by using (7):

$$\gamma = \frac{z(1+\rho_s)}{2\sqrt{N-1}},$$

if the suggestions regarding the form of the dispersion σ_0 and the approximating function $\rho(\tau)$ are correct. In the opposite case, by following the suggested plan it is possible to obtain a formula similar to (7). Use of expression (7) itself is justified in a first approximation. A graph of formula (7) at probability $p(a) = 0.9$ and various $\tau_{0,5}$ is given in Figure 5 and does not require special explanation.

For correct use of the results attention must be given to the fact that maximum and minimum dimensions were obtained above which can be determined in the direction of the base line. The entire measuring system altogether is capable of registering the dimensions of heterogeneities less than $2.8S_{min}$ (S_{min} is the minimum base).

The point is that the value found above for Δ_{min} corresponds to $V'_{c\,min}$ in the direction of the base line.

But the minimum value of V'_c that can be registered by the measuring system will depend on the relative location of the observation points and is determined in the direction of the smallest height of the "measuring triangle", $S_0 = S_1 \cos \alpha$ (see Figure 2).

Hence the minimum dimension measurable by the system as a whole is

$$\Delta^0_{min} = \Delta_{min} \cos \alpha.$$

If the ionospheric heterogeneities have substantial anisotropy of form and comparatively stable orientation of the axes of anisotropy, then it is expedient to take such observation points that the distances between them will not be the same and the largest base would be orientated parallel to the axis of the heterogeneities. Such a disposition of the "measuring triangle" enables measuring the large dimensions of the heterogeneities on the large base and the small on the small, which assures more advantageous conditions of measurement on the score of accuracy.

In particular, the experimental investigations of large-scale heterogeneities revealed substantial anisotropy of form and predominant orientation of the axis of anisotropy along the magnetic meridian [3]. It is reasonable to assume that the above is also correct for small-scale heterogeneities [9,10].

The height of the "measuring triangle" should therefore be selected with consideration for the most often encountered values of the anisotropic index:

$$e = \frac{V'_{cmax}}{V'_{cmin}}.$$

According to the experimental data [3], the largest number of values of e lies in the limits of 1 - 7. The most frequently encountered value of e is of the order of 1.5-2. Hence, to assure the same accuracy in determining the dimensions of the heterogeneities in different directions one should select $S_0 = 1/eS_{max} \approx 0.5S_{max}$

(S_{max} is the largest base).

Thus the most convenient disposition of the observation points will be one in which they form an isosceles rectangular triangle whose hypotenuse is orientated parallel to the large axis of the heterogeneities. Such a measuring system will be capable of registering the horizontal dimensions of the heterogeneities lying, say, within the limits

$$2{,}8S_0 \leqslant \Delta \leqslant 46S_0.$$

It can be said that in investigating small-scale heterogeneities the observation interval $T \approx 5$ minutes, which at a minimum value of the radius of correlation for these heterogeneities is of the order of 1-2 seconds, enables obtaining a value of $N_{max} \approx 150$, which is close to that used in our work (in conformity with the large-scale heterogeneities $N_{max} \approx 120$). Further, the regions of the main values of $\rho(\tau)$ for fine and coarse heterogeneities are determined identically from the point of view of correlation analysis. Taking what has been said into consideration, and likewise the fact that the anisotropic index for fine heterogeneities is also of the order of 1.5-2 [10], all the recommendations regarding the geometry of the "measuring triangle" are correct both for coarse and for fine heterogeneities of the ionosphere.

Evaluation of Some of the Results of the Experiment

In conducting the experiment the three observation points were disposed as shown in Figure 2. The smallest height S_0 of the "measuring triangle" was about 16.8 km. A measuring system using a triangle with those dimensions is capable of measuring heterogeneities with a dimension of not less than $\Delta_{min} \approx 47$ km and not more than $\Delta_{max} \approx 1400$ km with an accuracy of 15%. In Figure 6, the distributions of the maximum (Fig.6a) and minimum (Fig. 6b) dimensions of the heterogeneities encountered in carrying out the experiment are given. The distributions given in Figure 6 are not uniform. The drops observed along the edges of the distributions are not connected with

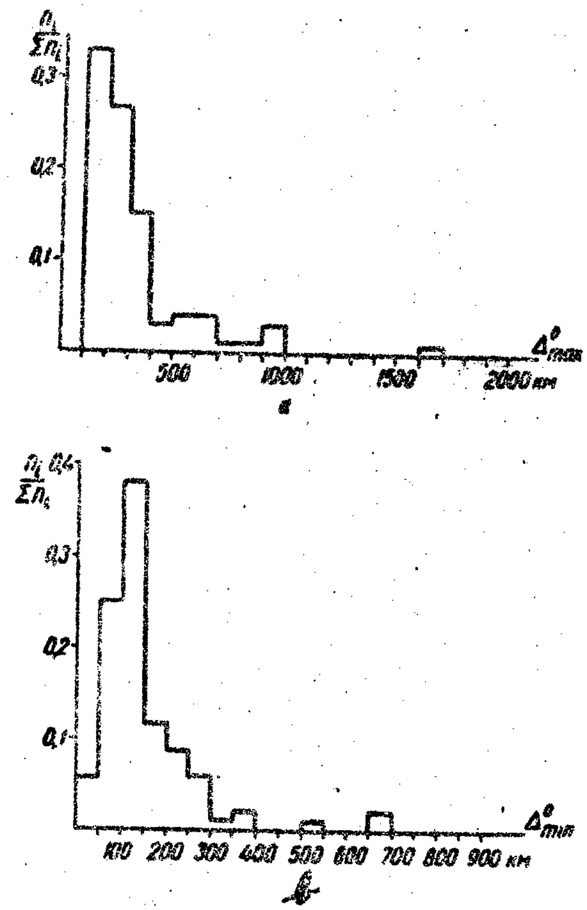

Fig. 6. Histograms of the horizontal dimensions of the heterogeneities: a - maximum; b - minimum.

the formation of the spectrum of the dimensions by the "resolving capacity" of the measuring system.

Thus the distributions obtained are the spectrum of the dimensions of the large scale heterogeneities in layer F_2 of the ionosphere. Using the results of the present work we evaluated the accuracy of the determination of the dimensions of the heterogeneities.

Figure 7 gives curves of the integral distribution of accuracy γ at probability $\rho(\alpha) = 0.9$ for three

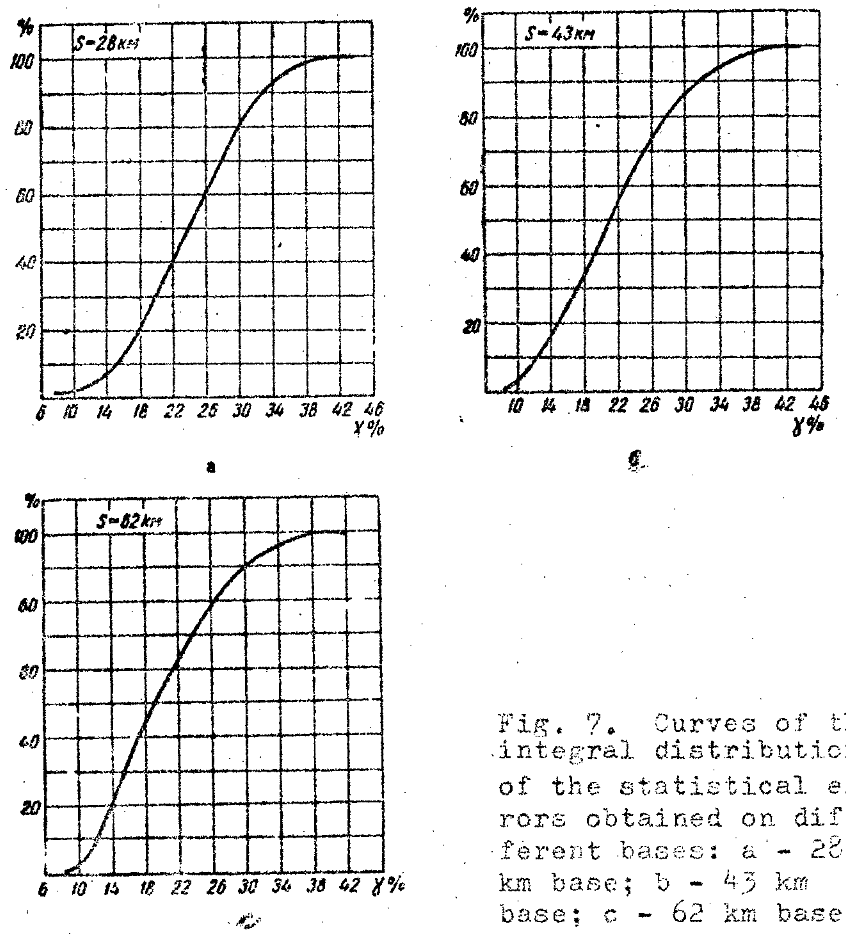

Fig. 7. Curves of the integral distribution of the statistical errors obtained on different bases: a - 28 km base; b - 43 km base; c - 62 km base.

different basic distances. It follows from these graphs that an accuracy of $\gamma \leq 30\%$ was assured on the 28, 43 and 62 km bases in 80, 86 and 90% of the cases respectively. These figures show that the distribution of the observation points used and the selected length of the observation interval, $T = 4$ hours, assured acceptable accuracy of the determinations of a rather wide (47-1400 km) spectrum of dimensions of the heterogeneities in an overwhelming majority of the cases.

Conclusions

1. The minimum horizontal dimension of ionospheric heterogeneities which can be measured on a base equal to S is 2.8 S, and the maximum is 23 S.

2. In investigations of the horizontal dimensions of heterogeneities it is expedient to dispose the observation points in such a way that they form an isosceles rectangular triangle whose hypotenuse is orientated parallel to the large axis of the heterogeneities (in the investigation of large-scale heterogeneities, parallel to the magnetic meridian).

3. With such a disposition of the observation points the horizontal dimensions of heterogeneities lying within the limits of

$$2{,}8S_0 \div 46S_0,$$

can be registered, where S is the smallest height of the "measuring triangle".

4. The spectrum of large-scale (47-1400 km) heterogeneities in the F_2 layer of the ionosphere is essentially irregular.

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